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LETTER TO THE EDITOR

On solutions of the equation $\nabla \times \boldsymbol{a} = k\boldsymbol{a}$

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Abstract. In a recent letter, Salingaros claimed that the equation curl $\boldsymbol{a} = k\boldsymbol{a}$ cannot be used to describe a magnetic field $\boldsymbol{B} = \boldsymbol{a}$. He bases this on the assertion that the magnetic field $\boldsymbol{B}$, caused by a current density $\boldsymbol{J}$, must be orthogonal to $\boldsymbol{J}$. This assertion is shown to be incorrect.

The vector differential equation

$$\nabla \times \boldsymbol{a} = k\boldsymbol{a} \tag{1}$$

has been the subject of recent controversy in this journal. Salingaros (1986a) has presented several reasons why this equation is mathematically inconsistent or physically incorrect. Maheswaran (1986) pointed out that several of these reasons are incorrect. In a second letter, Salingaros (1986b) reiterated the above reasons and presented a new argument against equation (1), this being for the case of a magnetic field $\boldsymbol{B}(\boldsymbol{r})$ obeying

$$\nabla \times \boldsymbol{B} = k\boldsymbol{B}. \tag{2}$$

Here $\boldsymbol{B}$ is the magnetic field caused by a current density $\boldsymbol{J}(\boldsymbol{r})$. Since $\boldsymbol{J} = \nabla \times \boldsymbol{B} = k\boldsymbol{B}$, it follows that $\boldsymbol{B}$ is parallel to $\boldsymbol{J}$. Salingaros then obtains a contradiction by arguing that $\boldsymbol{B}$ is orthogonal to $\boldsymbol{J}$. The purpose of this letter is to show that this new argument is also incorrect.

In his development Salingaros (1986b) asserts that a magnetic field $\boldsymbol{B}(\boldsymbol{r})$, caused by some current density $\boldsymbol{J}(\boldsymbol{r})$, must be (locally) orthogonal to $\boldsymbol{J}(\boldsymbol{r})$. He bases this on the fact that $\mathrm{d}\boldsymbol{B}$ (at some field point) caused by a current element $i\mathrm{d}\boldsymbol{l}$ (at some other source point) is orthogonal to $\mathrm{d}\boldsymbol{l}$. While this latter statement is true (from the Biot-Savart law), it does *not* follow that the magnetic field $\boldsymbol{B}(\boldsymbol{r})$ caused by a current density $\boldsymbol{J}(\boldsymbol{r})$ is (locally) orthogonal to $\boldsymbol{J}(\boldsymbol{r})$. The reason for this is that orthogonality does not obey a superposition principle.

Suppose there *were* some pair $(\boldsymbol{B}, \boldsymbol{J})$ which were locally orthogonal. A concrete example of this is furnished by a uniform azimuthal current density $\boldsymbol{J}(\rho, \phi, z) = C\boldsymbol{e}_\phi$ within the annular cylinder $a < \rho < b$, $0 < z < L$; here $\boldsymbol{B}$ has only $\boldsymbol{e}_\rho$ and $\boldsymbol{e}_z$ components. Now let $(\boldsymbol{B}', \boldsymbol{J}')$ be any other orthogonal pair (which could be simply the first pair $(\boldsymbol{B}, \boldsymbol{J})$ just rotated or translated slightly). Then, considering the superposition of the two problems, $\boldsymbol{B}'' = \boldsymbol{B} + \boldsymbol{B}'$ is obviously *not* necessarily locally orthogonal to $\boldsymbol{J}'' = \boldsymbol{J} + \boldsymbol{J}'$.

References

Maheswaran M 1986 *J. Phys. A: Math. Gen.* **19** L761-2
Salingaros N A 1986a *J. Phys. A: Math. Gen.* **19** L101-4
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